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2016 Mathematics

Advanced Higher

Finalised Marking Instructions

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General Marking Principles for Advanced Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x = 1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2 + 5x + 7 = 9x + 4$ $x - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 1$ or 3

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark. Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.	(a)		•[1] evidence of use of product rule [1,2]	•[1] $(\ldots)\tan^{-1}2x + x(\ldots)$	3
			•[2] one resultant term of the product correct	•[2] $1.\tan^{-1}2x$ or $x.\dfrac{1}{1+(2x)^2}.2$	
			•[3] complete differentiation [3]	•[3] $\tan^{-1}2x + \dfrac{2x}{1+4x^2}$	

Notes:

1. Evidence for the award of •[1] should take the form $f(x)\times(\ldots)+g(x)\times(\ldots)$.
2. For a candidate who interprets $\tan^{-1}2x$ as $(\tan 2x)^{-1}$ •[3] is not available.
3. Accept $(2x)^2$ when awarding •[3].

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[4] evidence of use of quotient or product rule and one term of numerator correct	•[4] $(-2x)(1+4x^2)-$	3
			•[5] complete differentiation correctly	•[5] $\dfrac{\ldots(1-x^2).8x}{(1+4x^2)^2}$	
			•[6] simplify answer [4,5]	•[6] $-\dfrac{10x}{(1+4x^2)^2}$ or $\dfrac{-10x}{(1+4x^2)^2}$	

Notes:

4. Where a candidate uses the product rule, simplification to $-\dfrac{10x}{(1+4x^2)^2}$ **or** $-10x(1+4x^2)^{-2}$ will be required in order to obtain •[6].
5. Incorrect working subsequent to a correct answer should be penalised in this instance eg an incorrect expansion of the denominator.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		•[7] correct derivatives	•[7] 6 and $\sin t$	2
			•[8] find $\dfrac{dy}{dx}$	•[8] $\dfrac{1}{6}\sin t$	

Notes

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.	(a)		•¹ interpret geometric series •² evidence of strategy [1,2] •³ value [2]	•¹ $ar = 108$ and $ar^4 = 4$ •² $\frac{ar^4}{ar}$ $r^3 = \frac{1}{27}$ •³ $r = \frac{1}{3}$	3

Notes:

1. For •² accept $r^3 = \frac{1}{27}$.
2. For a statement of the answer only, award •¹ and •³. To earn •² there must be evidence of a strategy eg $108 \to 36 \to 12 \to 4$ gives $r = \frac{1}{3}$.

Commonly Observed Responses:

	(b)		•⁴ know condition [3,4]	•⁴ $-1 < \frac{1}{3} < 1$	1

Notes:

3. For •⁴ $\frac{1}{3}$ may be replaced with a letter consistent with their answer to (a). However, in the case where a candidate obtains a value in (a) outside the open interval $(-1, 1)$ •⁴ will only be available where they also acknowledge that there is no sum to infinity.
4. Only award •⁴ for a strict inequality, whether it is expressed algebraically or in words.

Commonly Observed Responses:

	(c)		•⁵ calculate the first term •⁶ value [5,6]	•⁵ $a = 324$ •⁶ $\frac{324}{1-\frac{1}{3}}$ or equivalent leading to 486	2

Notes:

5. For an incorrect value in (a) •⁶ will only be available provided the value satisfies the condition for convergence.
6. Where a candidate has used $S_\infty = \frac{a(1-r^\infty)}{1-r}$ full credit is available.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.			•¹ state general term [2]	•¹ ${}^{13}C_r\left(\frac{3}{x}\right)^{13-r}\left(-2x\right)^r$	5
			•² simplify powers of x **OR** coefficients **and** signs [2,5]	•² $(3)^{13-r}(-2)^r$ or x^{2r-13}	
			•³ state simplified general term (completes simplification) [2,5]	•³ ${}^{13}C_r(3)^{13-r}(-2)^r x^{2r-13}$	
			•⁴ determine value of r [3,4]	•⁴ $2r-13=9 \Rightarrow r=11$	
			•⁵ evaluate term [1,3]	•⁵ $-1437696x^9$	

Notes:

1. Accept -1437696.
2. For •¹ accept the initial appearance of $\sum_{r=0}^{13} {}^{13}C_r\left(\frac{3}{x}\right)^{13-r}\left(-2x\right)^r$ as bad form. •² and •³ are available only to candidates who simplify a general term correctly.
3. •⁴ and •⁵ are the only marks available to candidates who have not proceeded from a general term eg. an expansion using Pascal's Triangle. The required term must be explicitly identified in order for •⁵ to be awarded.
4. Starting with ${}^{13}C_r\left(\frac{3}{x}\right)^{r}\left(-2x\right)^{13-r}$ leading to $r=2$ can also gain full credit.
5. Accept $\frac{1}{x^{13-2r}}$ when awarding •² or •³.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.			•[1] Construct augmented matrix	•[1] $\left(\begin{array}{ccc\|c} 1 & 2 & 3 & 3 \\ 2 & -1 & 4 & 5 \\ 1 & -3 & 2\lambda & 2 \end{array}\right)$	4
			•[2] Use row operations to establish first two zero elements [1]	•[2] $\left(\begin{array}{ccc\|c} 1 & 2 & 3 & 3 \\ 0 & 5 & 2 & 1 \\ 0 & -5 & 2\lambda-3 & -1 \end{array}\right)$	
			•[3] Establish third zero element **OR** recognise linear relationship between two rows [1,2]	•[3] $\left(\begin{array}{ccc\|c} 1 & 2 & 3 & 3 \\ 0 & 5 & 2 & 1 \\ 0 & 0 & 2\lambda-1 & 0 \end{array}\right)$ **OR** $2\lambda-3=-2$	
			•[4] State value of λ [2]	•[4] $\lambda=\frac{1}{2}$	

Notes:

1. Elementary row operations must be carried out correctly for •[2] and •[3] to be awarded.
2. •[4] is only available where a candidate's final matrix exhibits redundancy.
3. Disregard any working/statement subsequent to $\lambda=\frac{1}{2}$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			•[1] show true for $n=1$ [1]	•[1] LHS: $1(3-1)=2$ RHS: $1^2(1+1)=2$ So true for $n=1$	**4**
			•[2] assume true for $n=k$ [2] **AND** consider $n=k+1$	•[2] $\sum_{r=1}^{k} r(3r-1)=k^2(k+1)$ **and** $\sum_{r=1}^{k+1} r(3r-1)=$ $\ldots=\sum_{r=1}^{k} r(3r-1)\ +\ (k+1)(3(k+1)-1)$	
			•[3] correct statement of sum to $(k+1)$ terms using inductive hypothesis	•[3] $=k^2(k+1)+(k+1)(3k+2)$ $=(k+1)\left[k^2+3k+2\right]$ $=(k+1)(k+1)(k+2)$	
			•[4] express explicitly in terms of $(k+1)$ **or** achieve stated aim/goal [3,4] **AND** communicate	•[4] $=(k+1)^2((k+1)+1)$, thus if true for $n=k$ then true for $n=k+1$ but since true for $n=1$, then by induction true for all $n\in\mathbb{N}$	

Notes:

1. "RHS = 2, LHS = 2" and/or "True for $n=1$" are insufficient for the award of •[1]. A candidate must demonstrate evidence of substitution into both expressions.

2. For •[2] acceptable phrases include: "If true for..."; "Suppose true for…"; "Assume true for…". However, ***not*** acceptable: "Consider $n=k$", "assume $n=k$" and "True for $n=k$".
Allow if appears at conclusion.

3. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently achieve the stated aim/goal.

4. Minimum acceptable form for •[4]: "Then true for $n=k+1$, but since true for $n=1$, then true for all n" or equivalent.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			**Method 1**		6
			•[1] for either function: first derivative and two evaluations **OR** all three derivatives **OR** all four evaluations	•[1] $f(x)=\sin 3x$ $f(0)=0$ $f'(x)=3\cos 3x$ $f'(0)=3$ $f''(x)=-9\sin 3x$ $f''(0)=0$ $f'''(x)=-27\cos 3x$ $f'''(0)=-27$ $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3\ldots$	
			•[2] complete derivatives and evaluations **AND** substitute	•[2] $f(x)=3x-\frac{27}{3!}x^3$ $=3x-\frac{9}{2}x^3$	
			•[3] for second function: first derivative and two evaluations **OR** all three derivatives **OR** all four evaluations	•[3] $f(x)=e^{4x}$ $f(0)=1$ $f'(x)=4e^{4x}$ $f'(0)=4$ $f''(x)=16e^{4x}$ $f''(0)=16$ $f'''(x)=64e^{4x}$ $f'''(0)=64$	
			•[4] complete derivatives and evaluations **AND** substitute	•[4] $f(x)=1+4x+\frac{16x^2}{2}+\frac{64x^3}{6}$ $=1+4x+8x^2+\frac{32}{3}x^3$	
			•[5] multiply expressions	•[5] $e^{4x}\sin 3x=\left(3x-\frac{9}{2}x^3\ldots\right)\left(1+4x+8x^2+\frac{32}{3}x^3\ldots\right)$ $=24x^3-\frac{9}{2}x^3+12x^2+3x\ \ldots$	
			•[6] multiply out and simplify Note 2	•[6] $=3x+12x^2+\frac{39}{2}x^3\ \ldots$	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

1. If a candidate chooses to use the product rule to obtain the Maclaurin series for $e^{4x}\sin 3x$ without first obtaining series for e^{4x} and $\sin 3x$ separately then only $\bullet^5$ and $\bullet^6$ are potentially available. In this instance for the award of $\bullet^5$ apply the same principle as that used to award $\bullet^1$ and $\bullet^3$.

$f(x)=e^{4x}\sin 3x$ $\quad$ $f(0)=0$

$f'(x)=4e^{4x}\sin 3x+3e^{4x}\cos 3x$ $\quad$ $f'(0)=3$

$f''(x)=7e^{4x}\sin 3x+24e^{4x}\cos 3x$ $\quad$ $f''(0)=24$

$f'''(x)=-44e^{4x}\sin 3x+117e^{4x}\cos 3x$ $\quad$ $f'''(0)=117$

2. At $\bullet^6$ the appearance of terms in x^4 or above should be disregarded.

Commonly Observed Responses:

			Generic Scheme	Illustrative Scheme	
			<u>**Method 2**</u>		
			$\bullet^1$ state the Maclaurin expansion for $\sin x$	$\bullet^1$ $\sin x = x-\frac{x^3}{3!}...$	1
			$\bullet^2$ substitute	$\bullet^2$ $\sin 3x = 3x-\frac{(3x)^3}{3!}...$ $\sin 3x = 3x-\frac{9x^3}{2}...$	
			$\bullet^3$ state the Maclaurin expansion for e^x	$\bullet^3$ $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}...$	1
			$\bullet^4$ substitute	$\bullet^4$ $e^{4x}=1+4x+\frac{(4x)^2}{2!}+\frac{(4x)^3}{3!}...$ $e^{4x}=1+4x+8x^2+\frac{32x^3}{3}...$	
			$\bullet^5$ multiply expressions	$\bullet^5$ $e^{4x}\sin 3x=\left(1+4x+8x^2...\right)\left(3x-\frac{9x^3}{2}...\right)$	
			$\bullet^6$ multiply out and simplify	$\bullet^6$ $e^{4x}\sin 3x=3x+12x^2+\frac{39x^3}{2}+....$	

Notes:

1. For a candidate who writes down $\sin 3x = 3x-\frac{(3x)^3}{3!}...$ without first writing down the series for $\sin x$ then $\bullet^1$ may be awarded. A similar principle may be applied to the awarding of $\bullet^3$ if required.
2. At $\bullet^6$ the appearance of terms in x^4 or above should be disregarded.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		•[1] calculate determinant [1]	•[1] -2	1

Notes:

1. If a candidate chooses to find A^{-1} then •[1] is only available where '$\det A$' is clearly identified.

Commonly Observed Responses:

$A^{-1} = \dfrac{1}{\det A}(\ldots)$

$A^{-1} = \dfrac{1}{-2}(\ldots)$

Award •[1]

$A^{-1} = \dfrac{1}{-2}(\ldots)$

Do not award •[1]

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		**Method 1**		3
			•[2] find A^2	•[2] $A^2 = \begin{pmatrix} 4 & 0 \\ \lambda & 1 \end{pmatrix}$	
			•[3] use an appropriate method	•[3] $A^2 = \begin{pmatrix} 2 & 0 \\ \lambda & -1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $A^2 = A + 2I$	
			•[4] write in required form and explicitly state values of *p* and *q* Note 1	•[4] $p = 1$ **and** $q = 2$	
			Method 2		
			•[2] find A^2	•[2] $A^2 = \begin{pmatrix} 4 & 0 \\ \lambda & 1 \end{pmatrix}$	
			•[3] use an appropriate method	•[3] $A^2 = p\begin{pmatrix} 2 & 0 \\ \lambda & -1 \end{pmatrix} + q\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	
			•[4] write in required form and explicitly state values of *p* and *q* Note 1	•[4] $A^2 = A + 2I$ p = 1 **and** q = 2	

Notes:

1. $\begin{pmatrix} 4 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ \lambda & -1 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is acceptable for •[4] provided the values of p and q are explicitly stated.

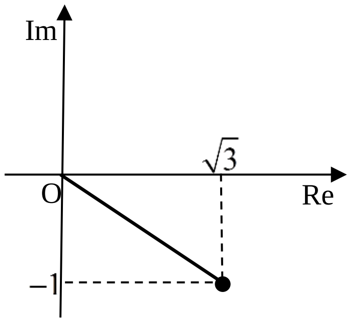
Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		•[5] square expression found in (b) 1,2,3 •[6] substitute for A^2 and complete process	•[5] $A^4 = (A+2I)^2$ $= A^2 + 4AI + 4I^2$ $= A + 2I + 4A + 4I$ •[6] $= 5A + 6I$	**2**

Notes:

1. •[5] may be obtained by squaring $\begin{pmatrix} 4 & 0 \\ \lambda & 1 \end{pmatrix}$ to give $\begin{pmatrix} 16 & 0 \\ 5\lambda & 1 \end{pmatrix}$ and identifying the coefficient of A as 5. This leads to •[6] using the same method as in (b).
2. Accept equivalent expressions eg $= A^2 + 4A + 4I$.
3. Candidates may calculate A^3 first so •[5] can be awarded for $A^3 = 3A + 2I$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•[1] correctly plot z on Argand diagram [1,2,3,4]	•[1] 	**1**

Notes:

1. Do not penalise the omission of the diagonal line.
2. Treat alternative axis labels as bad form (to include the case where there are no labels).
3. Accept a point labelled using coordinates: $(\sqrt{3}, -1)$ and, in this instance, $(\sqrt{3}, -i)$.
4. The minimum acceptable response for the award of •[1] is a point in quadrant 4 together with $\sqrt{3}$ and -1 (or $-i$).

Commonly Observed Responses:

	(b)		•[2] find modulus or argument [1,2,3,6]	•[2] $\|w\| = 2a$ or $\arg(w) = -\frac{\pi}{6}$	**2**
			•[3] complete and express in polar form [3,4,5,6]	•[3] $w = 2a\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$	

Notes:

1. For the award of •[2] and •[3] accept any answer of the form $-\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$.
2. For the award of •[2] and •[3] accept any answer of the form $(-30 + 360k)°,\ k \in \mathbb{Z}$.
3. A candidate who chooses to work in degrees can only be awarded •[3] provided the degree symbol appears at some point within question 8.
4. Award •[3] for $w = 2a\left(\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right)$.
5. At •[3] do not accept $w = a\left[2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)\right]$.
6. Working subsequent to the appearance of $-\frac{\pi}{6}$ should be penalised where it leads to the use of an incorrect argument.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(c)		**Method 1**		**3**
			•[4] process modulus	•[4] $256a^8$	
			•[5] process argument [1,2,3,4,5]	•[5] $\ldots\left(\cos\left(-\frac{8\pi}{6}\right)+i\sin\left(-\frac{8\pi}{6}\right)\right)$	
			•[6] evaluate and express in form $ka^n\left(x+i\sqrt{y}\right)$	•[6] $w^8=128a^8\left(-1+i\sqrt{3}\right)$	

Notes:

1. For the award of •[5] accept any answer of the form $-\frac{4\pi}{3}+2k\pi,\ k\in\mathbb{Z}$.
2. For the award of •[5] accept any answer of the form $\left(-240+360k\right)^\circ,\ k\in\mathbb{Z}$.
3. A candidate who chooses to work in degrees can only be awarded •[5] provided the degree symbol appears at some point within question 8.
4. Do not penalise unsimplified fractions.
5. Award •[5] for $\ldots\left(\cos\frac{8\pi}{6}-i\sin\frac{8\pi}{6}\right)$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 2		**3**
			•[4] find w^2 correctly and attempt to find a higher power of w Note 1	•[4] eg $w^2=a^2\left(2-2i\sqrt{3}\right)$ and $w^3=a^2\left(2-2i\sqrt{3}\right)\times a\left(\sqrt{3}-i\right)$.	
			•[5] obtain w^4	•[5] $w^4=a^4\left(-8-8i\sqrt{3}\right)$	
			•[6] complete expansion and express in form $ka^n\left(x+i\sqrt{y}\right)$	•[6] $w^8=128a^8\left(-1+i\sqrt{3}\right)$	

Notes:

1. Accept the omission of 'a' at •[4] and •[5] provided a^8 appears in the final answer.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 3		**3**
			•[4] write down full binomial expansion [1,2]	•[4] $\binom{8}{0}\left(\sqrt{3}\right)^8(-i)^0+\binom{8}{1}\left(\sqrt{3}\right)^7(-i)^1+\binom{8}{2}\left(\sqrt{3}\right)^7(-i)^2\ldots+\binom{8}{8}\left(\sqrt{3}\right)^0(-i)^8$	
			•[5] simplifies individual terms	•[5] $81-216i\sqrt{3}-756+504i\sqrt{3}+630-168i\sqrt{3}-84+8i\sqrt{3}+1$	
			•[6] complete expansion and express in form $ka^n\left(x+i\sqrt{y}\right)$	•[6] $w^8=128a^8\left(-1+i\sqrt{3}\right)$	

Notes:

1. For the award of •[4] a **full** expansion must be written out.
2. Accept the omission of 'a' at •[4] and •[5] provided a^8 appears in the final answer.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			•1 know to use integration by parts **and** start process 1,2,3	•1 $\frac{1}{8}x^8(\ln x)^2 - \ldots$	6
			•2 correct choice of functions to differentiate and integrate **AND** application thereof 1,2,3	•2 $\ldots - \frac{1}{8}\int x^8 \times \frac{d}{dx}\left((\ln x)^2\right)dx$	
			•3 differentiate $(\ln x)^2$ 4	•3 $\frac{1}{8}x^8(\ln x)^2 - \frac{1}{4}\int x^7(\ln x)\,dx$	
			•4 know to use second application and begin process 1,2,3,4	•4 $\ldots - \left[\frac{1}{32}x^8(\ln x) - \frac{1}{32}\int x^8\left(\frac{1}{x}\right)dx\right]$	
			•5 complete second application	•5 $\ldots - \left[\frac{1}{32}x^8(\ln x) - \frac{1}{256}x^8\right]$	
			•6 simplify 5	•6 $\frac{1}{8}x^8(\ln x)^2 - \frac{1}{32}x^8(\ln x) + \frac{1}{256}x^8 + c$	

Notes:

1. For candidates who attempt to integrate $(\ln x)^2$ and differentiate x^7 then •1, •4 and •6 may be awarded but not •2, •3 and •5.
2. Evidence of use of integration by parts would be the appearance of an attempt to integrate one term and differentiate the other.
3. For candidates who attempt to substitute for $\ln x$ eg $t = \ln x$ leading to $\int t^2 e^{8t}\,dt$ then
   - •1 becomes available upon evidence of using integration by parts ie. $t^2.\frac{1}{8}e^{8t} - \ldots$
   - •6 is only available for a final answer expressed as a function of *x*.
4. For candidates who incorrectly differentiate $(\ln x)^2$ and do not require a second application of integration by parts, only •1, •2 and •6 are available.
5. Do not penalise the omission of "$+c$".

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.			•[1] give counterexample	•[1] eg. choose $p=7$ $2(7)+1=15$ and since $15=5\times3$, hence not prime, statement is false	4
			•[2] set up n Notes 1,2	•[2] $n=3a+1$, $a\in\mathbb{N}_0$	
			•[3] consider expansion of n^3 Note 3	•[3] $n^3=27a^3+27a^2+9a+1$	
			•[4] complete proof with conclusion 4	•[4] $=3\left(9a^3+9a^2+3a\right)+1$ and statement such as "so n^3 has remainder 1 when divided by 3 ∴ statement is true".	

Notes:

1. Do not penalise the omission of $a\in\mathbb{N}_0$ in •[2].
2. Treat a statement such as $n=3n+1$ as bad form.
3. •[3] can only be awarded for the correct expansion of $(3a+1)^3$.
4. Minimum statement of conclusion in •[4] is "true".
5. Where a candidate invokes an incorrect use of proof by contradiction full credit may still be available provided all relevant steps are included.

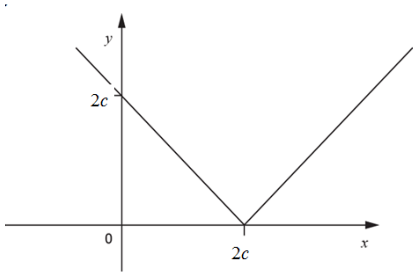
Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.			**Method 1**		4
			•[1] state differential equation [1,2]	•[1] $\frac{dh}{dt}=5$	
			•[2] state relationship or apply chain rule [3]	•[2] $\frac{dV}{dt}=\frac{dV}{dh}\cdot\frac{dh}{dt}$ $V=h^3$	
			•[3] find the rate of change of volume with respect to height [3]	•[3] $\frac{dV}{dh}=3h^2$	
			•[4] evaluate [4]	•[4] $\frac{dV}{dt}=3h^2\times5=3(3)^2\times5=135\ \text{cm}^3\,\text{s}^{-1}$	
			Method 2		
			•[1] express volume as a function of time	•[1] $V=125t^3$	
			•[2] find rate of change of volume with respect to time	•[2] $\frac{dV}{dt}=375t^2$	
			•[3] find value of *t*	•[3] $t=\frac{3}{5}$	
			•[4] evaluate	•[4] $\frac{dV}{dt}=375\left(\frac{3}{5}\right)^2=135\ \text{cm}^3\,\text{s}^{-1}$	

Notes:

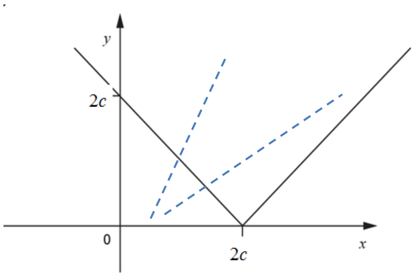
1. A candidate who assumes that only the height changes – and that the length and breadth are constant – can be awarded •[1] and •[2] only.
2. Where a candidate uses the wrong formula for the volume of a cube only •[1] and •[2] are available.
3. A candidate using Method 1 who writes $\frac{dV}{dt}=3h^2\frac{dh}{dt}$ can be awarded •[2] and •[3].
4. To award •[4] there must be evidence of substituting 3 and 5. Correct units must also be included.

Commonly Observed Responses:

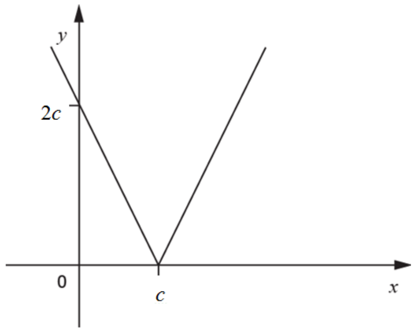
Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		•[1] correct shape •[2] graph passes through $2c$ on the positive x- and y-axes	•[1,2] 	**2**

Notes:

1. To award •[2], the second arm must be sketched to within 15° of the reflected angle.

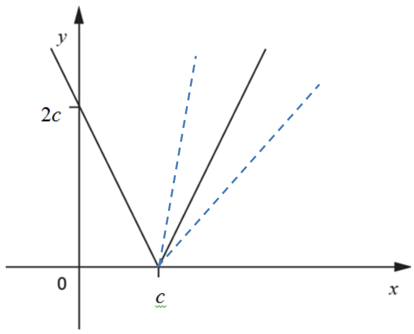


Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[3] graph of $y = \|2f(x)\|$ passing through $2c$ on the positive y-axis [1] •[4] correct shape (symmetrical V) meeting positive x-axis at c [2]	•[3,4] 	2

Notes:

1. For a candidate who sketches the graph of $y = 2f(x)$ award •[3] for showing a straight line passing through $(0, -2c)$.
2. To award •[4], the second arm must be sketched to within 15° of the reflected angle.



Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.			•[1] correct application of partial fractions	•[1] $\frac{3x+32}{(x+4)(6-x)}=\frac{A}{x+4}+\frac{B}{6-x}$	9
			•[2] starts process	•[2] $3x+32=A(6-x)+B(x+4)$	
			•[3] calculate one value	•[3] $A=2$	
			•[4] calculate second value	•[4] $B=5$	
			•[5] re-state integral in partial fractions	•[5] $\int_3^4\left(\frac{2}{(x+4)}+\frac{5}{(6-x)}\right)dx$	
			•[6] one term correctly integrated [1]	•[6] $\left[2\ln\lvert x+4\rvert \ldots\right.$	
			•[7] Integrate second term correctly [1]	•[7] $\left.\ldots-5\ln\lvert 6-x\rvert\,\right]_3^4$	
			•[8] substitute limits	•[8] $\left(2\ln\lvert 4+4\rvert-5\ln\lvert 6-4\rvert\right)$ $-\left(2\ln\lvert 3+4\rvert-5\ln\lvert 6-3\rvert\right)$	
			•[9] evaluate to expected form Note 3	•[9] $=\ln\frac{486}{49}$	

Notes:

1. Do not penalise lack of modulus signs unless the candidate attempts to integrate $\frac{1}{x-6}$ rather than $\frac{1}{6-x}$.
2. Award maximum [8/9] for appropriate working leading to $\ln\frac{98}{243}$ (•[9] lost)

   OR

   $\ln\frac{2048}{11907}$ (•[7] lost).
3. Do not penalise unsimplified fractions in •[9].

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		•[1] convert any two components of L_2 to parametric form [1]	•[1] two from $x = 3 - 2\mu$, $y = 8 + \mu$, $z = -1 + 3\mu$	**5**
			•[2] two linear equations involving two distinct parameters	•[2] two from $4 + 3\lambda = 3 - 2\mu$, $2 + 4\lambda = 8 + \mu$, $-7\lambda = -1 + 3\mu$	
			•[3] find parameter values	•[3] $\lambda = 1,\ \mu = -2$	
			•[4] verify third component in **both** equations or equivalent	•[4] eg $z_1 = -7 \times 1$ and $z_2 = 3(-2) - 1$ therefore the lines intersect	
			•[5] find point of intersection	•[5] $(7,\ 6,\ -7)$	

Notes:

1. A candidate who uses λ as the second parameter can only be awarded •[1] unless this is rectified later in the question.
2. Do not penalise the omission of the statement 'therefore the lines intersect'.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		•[6] identify first direction vector [1,2,3]	•[6] $\mathbf{d}_1 = 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$	4
			•[7] identify second direction vector [1,2,3]	•[7] $\mathbf{d}_2 = -2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$	
			•[8] calculate magnitudes and scalar product	•[8] $\lvert\mathbf{d}_1\rvert = \sqrt{74}$, $\lvert\mathbf{d}_2\rvert = \sqrt{14}$ and $\mathbf{d}_1 \cdot \mathbf{d}_2 = -6 + 4 - 21 = -23$	
			•[9] calculate obtuse angle [4,5]	•[9] $\cos^{-1}\left(\frac{-23}{\sqrt{74}\sqrt{14}}\right) \approx 135 \cdot 6°$	

Notes:

1. For $L_1 = 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ and $L_2 = -2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or equivalent, lose •[6] but •[7] is available (repeated error).
2. Do not penalise $\mathbf{L}_1 = 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ and $\mathbf{L}_2 = -2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.
3. For $L_1 : 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ and $L_2 : -2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or equivalent, •[6] and •[7] are both available.
4. For the award of •[9] accept $136°$.
5. •[9] is not available to candidates who calculate an obtuse angle correctly but who subsequently calculate an acute angle.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.			•[1] state auxiliary equation [1]	•[1] $m^2+5m+6=0$ $m=-3,\ m=-2$	**10**
			•[2] solve auxiliary equation and state complementary function [2,3]	•[2] $y=Ae^{-3x}+Be^{-2x}$	
			•[3] construct particular integral	•[3] $y=Cx^2+Dx+E$	
			•[4] differentiate particular integral	•[4] $\frac{dy}{dx}=2Cx+D$ and $\frac{d^2y}{dx^2}=2C$	
			•[5] calculate one coefficient of the particular integral	•[5] $C=2$	
			•[6] calculate remaining coefficients	•[6] $D=-3, E=1$ $y=Ae^{-3x}+Be^{-2x}+2x^2-3x+1$	
			•[7] differentiate general solution [3]	•[7] $\frac{dy}{dx}=-3Ae^{-3x}-2Be^{-2x}+4x-3$	
			•[8] construct equations using given conditions	•[8] $A+B=-7$ and $3A+2B=-6$ or equivalent	
			•[9] Find one coefficient	•[9] $A=8$ or $B=-15$	
			•[10] Find other coefficient **and** state particular solution	•[10] $y=8e^{-3x}-15e^{-2x}+2x^2-3x+1$	

Notes:

1. For •[1] do not penalise the omission of $'=0'$.
2. •[2] can be awarded if the Complementary Function appears later as part of the general solution, as opposed to being explicitly stated immediately after solving the Auxiliary Equation.
3. A candidate who obtains $m=2$ and $m=3$ from a correct auxiliary equation, leading to $y=20e^{3x}-27e^{2x}+2x^2-3x+1$ cannot gain •[2] but all other marks are available.
4. Where a candidate substitutes the given conditions into the Complementary Function to obtain values of A and B and then finds the particular integral correctly •[8] and •[9] are unavailable.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
16.			**Method 1** – working in minutes ($t=0$ at noon)		9
			•[1] construct integral equation Note 1	•[1] $\int \frac{1}{(T-T_F)}\,dT = \int -k\,dt$	
			•[2] integrate 2	•[2] $\ln(T-T_F) = -kt + c$	
			•[3] find constant, c	•[3] $\ln(9\cdot8-4) = -k(0)+c$ $c = \ln 5\cdot8$	
			•[4] substitute using given information 4	•[4] $\ln(6\cdot5-4) = -15k + \ln 5\cdot8$	
			•[5] find constant, k	•[5] $k = \frac{\ln 2\cdot5 - \ln 5\cdot8}{-15} = 0\cdot05610...$	
			•[6] substitute given condition	•[6] $\ln(25-4) = -0\cdot05610...t + \ln 5\cdot8$	
			•[7] know how to find time	•[7] $t = \frac{\ln 21 - \ln 5\cdot8}{-0\cdot05610...}$	
			•[8] calculate time	•[8] $t = -22\cdot93...$	
			•[9] state the time to the nearest minute 3	•[9] The liquid was placed in the fridge at 11:37 (am)	

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 2 - working in minutes ($t=0$ when $T=25$)		
			•[1] construct integral equation Note 1	•[1] $\int \frac{1}{(T-T_F)}\,dT = \int -k\,dt$	
			•[2] integrate 2	•[2] $\ln(T-T_F) = -kt + c$	
			•[3] find constant, c.	•[3] $\ln(25-4) = -k(0)+c$, $c=\ln 21$	
			•[4] substitute using given information	•[4] $\ln(9\cdot8-4) = -k(t)+\ln 21$	
			•[5] know to use $t+15$ Note 5	•[5] appearance of $(t+15)$	
			•[6] use given condition	•[6] $\ln(6\cdot5-4) = -k(t+15)+\ln 21$	
			•[7] find constant, k Note 6	•[7] $k = -\frac{1}{15}\ln\left(\frac{2\cdot5}{5\cdot8}\right) = 0\cdot05610...$	
			•[8] calculate time	•[8] $t = \ln\left(\frac{21}{5\cdot8}\right) \div 0\cdot05610... = 22\cdot93$	
			•[9] state the time to the nearest minute 3	•[9] The liquid was placed in the fridge at 11:37 (am).	

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 3 – working in hours ($t=0$ at midnight)		
			•[1] construct integral equation Note 1	•[1] $\int \frac{1}{(T-T_F)}\,dT = \int -k\,dt$	
			•[2] integrate 2	•[2] $\ln\left(T-T_F\right) = -kt+c$	
			•[3] use initial conditions	•[3] $\ln 5\cdot 8 = -12k+c$	
			•[4] interpret later time	•[4] $\ln 2\cdot 5 = -12\cdot 25k+c$	
			•[5] find constant, k	•[5] $\ln 5\cdot 8 - \ln 2\cdot 5 = 0\cdot 25k$ $k = 3\cdot 366...$	
			•[6] find the constant, c	•[6] $\ln\left(9\cdot 8-4\right) = -3\cdot 366...\times 12+c$ $c = 42\cdot 15...$	
			•[7] know to find time	•[7] $\ln\left(25-4\right) = -3\cdot 366...t+42\cdot 15$	
			•[8] calculate time	•[8] $t = \frac{42\cdot 15-\ln 21}{3\cdot 366...}$ $= 11\cdot 62...$	
			•[9] state the time to the nearest minute 3	•[9] The liquid was placed in the fridge at 11:37 (am).	

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 4 – working in minutes ($t=0$ when $T=25$)		
			•[1] construct integral equation Note 1	•[1] $\int \frac{1}{(T-T_F)}dT = \int -k\,dt$	
			•[2] integrate [2]	•[2] $\ln(T-T_F) = -kt + c$ $T-T_F = e^{-kt+c}$ $T = Ae^{-kt} + T_F$ $T = Ae^{-kt} + 4$	
			•[3] use initial condition to calculate A	•[3] $25 = Ae^{-k(0)} + 4 \therefore A = 21$	
			•[4] substitute using given information	•[4] $9\cdot8 = 21e^{-kt} + 4$	
			•[5] know to use $t+15$ Note 7	•[5] appearance of $(t+15)$	
			•[6] substitute using given information	•[6] $6\cdot5 = 21e^{-k(t+15)} + 4$	
			•[7] find constant, k	•[7] $k = \frac{\ln\left(\frac{5\cdot8}{21}\right) - \ln\left(\frac{2\cdot5}{21}\right)}{15} = 0\cdot0561...$	
			•[8] calculate time	•[8] $t = \ln\left(\frac{21}{5\cdot8}\right) \div 0\cdot05610... = 22\cdot93...$	
			•[9] state the time to the nearest minute [3]	•[9] The liquid was placed in the fridge at 11:37 (am).	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

General note:

Many candidates may use a combination of the given methods. For all methods the evidence for $\bullet^1$, $\bullet^2$, $\bullet^8$ and $\bullet^9$ is the same. To award $\bullet^3$ up to $\bullet^7$ note that:

two marks are awarded for using two different values of T
one mark is awarded for finding the constant of integration
one mark is awarded for finding or eliminating k (refer to Note **6**)
one mark is awarded for dealing with the elapsed time (noon until 12:15)

1. Do not penalise the omission of integral symbols at $\bullet^1$. (All Methods)
2. Do not penalise omission of "$+c$" at $\bullet^2$. However, it is necessary to access some later marks. (All Methods)
3. Where a candidate obtains an incorrect final answer because of earlier rounding, only $\bullet^9$ is unavailable. (All Methods)
4. For Method 1, if the candidate works in hours:
   - $\bullet^4$ $\ln(6\cdot5-4) = -0\cdot25k + \ln(5\cdot8)$
   - $\bullet^5$ $k = -4(\ln 2\cdot5 - \ln 5\cdot8) = 3\cdot366...$
   - $\bullet^6$ $\ln(25-4) = -3\cdot366...t + \ln 5\cdot8$
   - $\bullet^7$ $t = \dfrac{\ln 21 - \ln 5\cdot8}{-3\cdot366...}$
   - $\bullet^8$ $t = -0\cdot3822...$
5. For Method 2, if the candidate works in hours:
   - $\bullet^5$ appearance of $(t+0\cdot25)$
   - $\bullet^6$ $\ln(6\cdot5-4) = -k(t+0\cdot25) + \ln 21$
   - $\bullet^7$ $k = -\dfrac{1}{0\cdot25}\ln\left(\dfrac{2\cdot5}{5\cdot8}\right) = 3\cdot366...$
   - $\bullet^8$ $t = \ln\left(\dfrac{21}{5\cdot8}\right) \div 0\cdot366... = 0\cdot3822...$
6. In Method 2 $\bullet^7$ can be awarded for eliminating k.
7. For Method 4, if the candidate works in hours:
   - $\bullet^5$ appearance of $(t+0\cdot25)$
   - $\bullet^6$ $6\cdot5 = 21e^{-k(t+0\cdot25)} + 4$
   - $\bullet^7$ $k = \dfrac{\ln\left(\dfrac{5\cdot8}{21}\right) - \ln\left(\dfrac{2\cdot5}{21}\right)}{0\cdot25} = 3\cdot366...$
   - $\bullet^8$ $t = \ln\left(\dfrac{21}{5\cdot8}\right) \div 3\cdot366... = 0\cdot3822...$

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]