National
Qualifications
2021
RESOURCE

2021 Mathematics
Paper 2

Advanced Higher

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

This is a transcription error and so the mark is not awarded.

$$x = 1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] differentiate	•[1] $f'(x) = 3\sec 2x \tan 2x \times 2$	2
			•[2] evaluate	•[2] $6\sqrt{2}$	

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•1 complete algorithm •2 equates gcd and evidence of substitution •3 a and b obtained	•1 $105 = 72 + 33$ $72 = 2 \times 33 + 6$ $33 = 5 \times 6 + 3$ $6 = 2 \times 3$ •2 $3 = 33 - 5 \times (72 - 2 \times 33)$ •3 $a = 11,\ b = -16$	**3**
	(b)		•4 find x and y	•4 $x = 1320,\ y = -1920$	**1**

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] use integration by parts and start process •[2] complete application •[3] complete integration	•[1] $\frac{1}{4}(2x+3)\sin 4x - \ldots$ •[2] $\ldots \int \frac{2}{4}\sin 4x\,dx$ •[3] $\frac{1}{4}(2x+3)\sin 4x + \frac{1}{8}\cos 4x + c$	3

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 start to find $\frac{dx}{dt}$	•1 $\frac{1}{\sqrt{1-(2t)^2}}$	3
			•2 complete $\frac{dx}{dt}$	•2 $\ldots\times 2$	
			•3 state $\frac{dy}{dt}$	•3 $\frac{1}{1+t^2}$	
	(b)		•4 begin process	•4 $(0,0)$ **OR** $\frac{dy}{dx}=\frac{1}{2}$	2
			•5 find the equation of the tangent	•5 $y=\frac{1}{2}x$	

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] find expression for A^4 in powers of two and less	•[1] $4A^2+20A+25I$ **OR** $9A^2+10A$	**2**
			•[2] substitute for A^2 and simplify	•[2] $28A+45I$	
	(b)		•[3] evidence of strategy	•[3] eg $A^{-1}A^2=2A^{-1}A+5A^{-1}I$ **OR** $A\left(A-2I\right)=5I$	**2**
			•[4] state expression	•[4] $\frac{1}{5}A-\frac{2}{5}I$	

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] find integrating factor •[2] write in correct form •[3] integrate right hand side including constant of integration •[4] find particular solution	•[1] e^{x^2} •[2] $e^{x^2}y = \int 14x\,dx$ •[3] $e^{x^2}y = 7x^2 + c$ •[4] $y = \dfrac{7x^2+3}{e^{x^2}}$	4

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] binomial expansion •[2] binomial coefficients and powers of 2 •[3] simplify	•[1] $\binom{3}{0}a^3+\binom{3}{1}a^2(2i)+\binom{3}{2}a(2i)^2+\binom{3}{3}(2i)^3$ Stated or implied by •[2] •[2] $a^3+6a^2i+12ai^2+8i^3$ •[3] $a^3-12a+(6a^2-8)i$	**3**
	(b)		•[4] substitute •[5] equate imaginary and real parts •[6] find a and b	•[4] $a^3-12a+(6a^2-8)i+3a+6i=b+148i$ •[5] $6a^2-2=148$ and $a^3-9a=b$ •[6] $a=5$, $b=80$	**3**

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)		•1 start to differentiate product with one term correct	•1 $2xy^3$ or $3x^2y^2\frac{dy}{dx}$	**4**
			•2 complete differentiation of product	•2 $3x^2y^2\frac{dy}{dx}$ or $2xy^3$	
			•3 differentiate remaining terms	•3 $\ldots 2e^{2y}\frac{dy}{dx}=0$	
			•4 write derivative explicitly in terms of x and y.	•4 $\frac{-2xy^3}{3x^2y^2+2e^{2y}}$	
	(b)		•5 express condition for stationary point	•5 $\frac{-2xy^3}{3x^2y^2+2e^{2y}}=0$	**3**
			•6 state corresponding values of both x and y.	•6 $x=0$ AND $y=0$	
			•7 show that there is one value of y when $x=0$ but no value of x when $y=0$	•7 $\left(0,\frac{1}{2}\ln 5\right)$ **AND** eg $1=5$ $\therefore$ no solution	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] write template •[2] find constants and express in partial fractions	•[1] $\frac{1}{x(5-x)} = \frac{A}{x} + \frac{B}{(5-x)}$ •[2] $\frac{1}{5x} + \frac{1}{5(5-x)}$	**2**

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		•3 write as integral equation •4 write LHS integral in partial fractions •5 integrate $\frac{1}{P}$ •6 complete integration •7 substitute values •8 evaluate constant of integration •9 take exponentials •10 write expression in terms of t	•3 $\int \frac{1}{P(5-P)}dP = \int \frac{1}{100}dt$ •4 $\frac{1}{5}\int\left(\frac{1}{P}+\frac{1}{(5-P)}\right)dP$ •5 $\frac{1}{5}(\ln P\ldots$ •6 $\frac{1}{5}(\ln P-\ln(5-P))=\frac{1}{100}t+c$ •7 $\frac{1}{5}(\ln 2\cdot5-\ln(5-2\cdot5))=\frac{1}{10}+c$ •8 $-\frac{1}{10}$ •9 $\frac{P}{5-P}=e^{0\cdot05t-0\cdot5}$ •10 $P=\frac{5e^{0\cdot05t-0\cdot5}}{1+e^{0\cdot05t-0\cdot5}}$	8

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.			•[1] show true for $n=2$	•[1] LHS: $\frac{1}{2(2-1)}=\frac{1}{2}$ RHS: $\frac{2-1}{2}=\frac{1}{2}$ so true for $n=2$	**5**
			•[2] assume (statement) true for $n=k$ **AND** consider whether (statement) true for $n=k+1$	•[2] suitable statement and $\sum_{r=2}^{k}\frac{1}{r(r-1)}=\frac{k-1}{k}$ **AND** $\sum_{r=2}^{k+1}\frac{1}{r(r-1)}=\ldots$	
			•[3] correct statement for sum to $(k+1)$ terms using inductive hypothesis	•[3] $\ldots=\frac{k-1}{k}+\frac{1}{(k+1)((k+1)-1)}$	
			•[4] express as a single fraction	•[4] $\frac{(k-1)(k+1)+1}{k(k+1)}$	
			•[5] express explicitly in terms of $(k+1)$ **AND** communicate	•[5] $\frac{k^2}{k(k+1)}$ leading to $\frac{k+1-1}{k+1}$ If true for $n=k$ then true for $n=k+1$. Also shown true for $n=2$ therefore, by induction, true for all $n\geq 2$.	

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)	(i)	•[1] state common difference	•[1] -6	**1**
		(ii)	•[2] calculate x	•[2] -4	**1**
	(b)	(i)	•[3] calculate first term	•[3] 115	**1**
		(ii)	•[4] state simplified expression	•[4] $121-6n$	**1**
	(c)		•[5] equate ratios of two pairs of consecutive terms •[6] rearrange into quadratic equation in standard form •[7] state values of and corresponding common ratios	•[5] eg $\frac{y-7}{y-1}=\frac{2y-9}{y-7}$ •[6] $y^2+3y-40=0$ •[7] $5,\ -\frac{1}{2}$ and $-8,\ \frac{5}{3}$	**3**
	(d)	(i)	•[8] state value of y, with justification	•[8] 5 and $\left\lvert -\frac{1}{2}\right\rvert<1$	**1**
		(ii)	•[9] form equation •[10] calculate a and state reason	•[9] $\frac{a}{1-\left(-\frac{1}{2}\right)}=\frac{64}{3}$ •[10] $a=32$ **and** eg No. This value would lead to -4, not 4, for the $y-1$ term	**1**

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] find two directed line segments	•[1] eg $\overrightarrow{AB}=\begin{pmatrix}2\\-5\\-4\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}-1\\4\\3\end{pmatrix}$	**4**
			•[2] begin to find vector product	•[2] eg $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -5 & -4\\ -1 & 4 & 3\end{vmatrix}$	
			•[3] calculate a normal vector	•[3] eg $\mathbf{n}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$ stated or implied at •[4]	
			•[4] obtain equation	•[4] $x-2y+3z=28$	
	(b)		•[5] state equation of π_2	•[5] $x-2y+3z=0$	**1**
	(c)	(i)	•[6] find parametric equations	•[6] $\begin{cases}x=4+t\\y=-2t\\z=8+3t\end{cases}$	**1**
	(c)	(ii)	•[7] substitute into equation of π_2	•[7] $4+t-2(-2t)+3(8+3t)=0$	**2**
			•[8] find coordinates of Q	•[8] $(2,4,2)$	

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)		•[1] express in appropriate form	•[1] $-1 = \cos\pi + i\sin\pi$	**1**
	(b)		•[2] verify root	•[2] $\cos\pi + \mathrm{i}\sin\pi$	**1**
	(c)		•[3] polar form	•[3] $z_2 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$	**1**
	(d)		•[4] any one from •[5] remaining roots	•[4,5] $\cos\left(-\frac{\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{\pi}{5}\right)$ or $\cos\left(-\frac{3\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{3\pi}{5}\right)$ or $\cos\pi + i\sin\pi$	**2**
	(e)		•[6] equate real part to zero •[7] complete proof	•[6] $\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} + \cos\left(-\frac{\pi}{5}\right) + \cos\left(-\frac{3\pi}{5}\right) - 1 = 0$ $2\cos\frac{\pi}{5} + 2\cos\frac{3\pi}{5} = 1$ •[7] **leading to** $\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}$	**2**

[END OF MARKING INSTRUCTIONS]