National
Qualifications
2024

2024 Mathematics Paper 2

Advanced Higher

Question Paper Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

This is a transcription error and so the mark is not awarded. [$x-4x+3=0$]

This is no longer a solution of a quadratic equation, so the mark is not awarded. [$x=1$]

The following example is an exception to the above

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded. [$x-4x+3=0$]

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any ‘Show that…’ question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] evidence of use of quotient rule with denominator and one term correct in the numerator [1,2] •[2] complete differentiation [2]	•[1] $\dfrac{7\cos 7x\left(1+x^2\right)-\ldots}{\left(1+x^2\right)^2}$ **OR** $\dfrac{\ldots-2x\sin 7x}{\left(1+x^2\right)^2}$ •[2] $\dfrac{7\cos 7x\left(1+x^2\right)-2x\sin 7x}{\left(1+x^2\right)^2}$	**2**

Notes:

1. Where a candidate equates y to the derivative, •[1] is still available.
2. For candidates who use the product rule, no marks may be awarded where an attempt to differentiate $\dfrac{1}{1+x^2}$ produces an inverse trigonometric function.

Commonly Observed Responses:

COR A
Product Rule

$7\cos 7x\left(1+x^2\right)^{-1}-\ldots$ or $\ldots-2x\sin 7x\left(1+x^2\right)^{-2}$ award •[1]

$7\cos 7x\left(1+x^2\right)^{-1}-2x\sin 7x\left(1+x^2\right)^{-2}$ award •[2]

COR B
Logarithmic Differentiation

$\ln y=\ln\sin 7x-\ln\left(1+x^2\right)$ AND $\dfrac{1}{y}\dfrac{dy}{dx}=\ldots$ award •[1]

$\dfrac{dy}{dx}=\dfrac{\sin 7x}{1+x^2}\left(\dfrac{7\cos 7x}{\sin 7x}-\dfrac{2x}{1+x^2}\right)$ award •[2]

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ complete algorithm [1]	•¹ $533 = 455 \times 1 + 78$ $455 = 78 \times 5 + 65$ $78 = 65 \times 1 + 13$ $(65 = 13 \times 5)$	3
			•² express gcd in terms of 455 and 533	•² eg $13 = (533 - 455 \times 1) \times 6 - 455$	
			•³ obtain a and b [2.3]	•³ $a = 6$, $b = -7$	

Notes:

1. At •¹ the gcd and the final line of working do not have to be stated explicitly.
2. The minimum requirement for •³ is $13 = 533 \times 6 + 455 \times (-7)$.
3. Do not accept $13 = 6 \times 533 - 7 \times 455$ where the values of a and b have not been explicitly stated.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] set up augmented matrix [1]	•[1] $\left(\begin{array}{ccc\|c} 1 & -1 & -3 & 1 \\ 2 & -3 & -5 & 8 \\ 1 & 2 & \lambda & -7 \end{array}\right)$	4
			•[2] obtain two zeros [2]	•[2] $\left(\begin{array}{ccc\|c} 1 & -1 & -3 & 1 \\ 0 & 1 & -1 & -6 \\ 0 & 3 & \lambda+3 & -8 \end{array}\right)$	
			•[3] complete row operations [2]	•[3] $\left(\begin{array}{ccc\|c} 1 & -1 & -3 & 1 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & \lambda+6 & 10 \end{array}\right)$	
			•[4] obtain expression for z [3]	•[4] $z = \dfrac{10}{\lambda+6}$	

Notes:

1. Where a candidate equates a 3×3 matrix to a 3×1 matrix, •[1] is not available. Otherwise, accept eg $x, y, z, =$ left in.
2. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •[2] and •[3].
3. Do not accept an answer of $(\lambda+6)z = 10$ when awarding •[4].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[5] state value of λ [1]	•[5] -6	1

Notes:

1. Do not award •[5] for $z = -6$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[6] find solution [1]	•[6] $x = 3,\ y = -4,\ z = 2$	1

Notes:

1. Where a candidate has made an error in (a), the value of x may be obtained from either the original equations or their final augmented matrix.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] state auxiliary equation [1] •[2] state general solution [2] •[3] differentiate •[4] solve for one constant •[5] find second constant and state particular solution [2]	•[1] $m^2 - 2m - 8 = 0$ •[2] $y = Ae^{-2x} + Be^{4x}$ •[3] $\frac{dy}{dx} = -2Ae^{-2x} + 4Be^{4x}$ •[4] $A = -5$ or $B = 3$ •[5] $y = -5e^{-2x} + 3e^{4x}$	5

Notes:

1. •[1] is not available where '$=0$' has been omitted.
2. Disregard the omission of '$y = \ldots$' at •[2].
3. Where a candidate does not give an expression for the derivative, •[3] may be awarded for $-2A + 4B = 22$.
4. Do not award •[5] if '$y = \ldots$' does not appear at that stage.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•1 state general term 1,2,3,4 •2 simplify powers of x **OR** •2 coefficients and signs 5 •3 complete simplification 5,6,7,8	•1 $\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$ •2 $2^{16-r}\left(-1\right)^r$ **OR** •2 x^{32-5r} •3 $\binom{16}{r}\left(-1\right)^r 2^{16-r}x^{32-5r}$	**3**

Notes:

1. Candidates may also proceed from $\binom{16}{r}\left(2x^2\right)^r\left(-\frac{1}{x^3}\right)^{16-r}$, leading to $\binom{16}{r}\left(-1\right)^{16-r}2^r x^{5r-48}$.
2. Where a candidate writes out a full expansion, •1, •2 and •3 are not available, unless the general term is identifiable in (b).
3. Where a candidate omits $\binom{16}{r}$, do not award •1.
4. Where a candidate does not fully substitute for n at first, •1 is available only where a correct expression in terms of x and r appears at a later stage.
5. Where a candidate produces a numerical power of 2 or $\left(-1\right)$, this must be evaluated for the award of the coefficient mark in the general term.
6. Award full marks if the expression at •3 appears without working.
7. Where a candidate in (a) produces an incorrect further simplification subsequent to the correct answer (eg $2^{16-r}\left(-1\right)^r$ becomes $\left(-2\right)^{16}$), •3 is not available.
8. Note that $\binom{16}{r}\left(-2\right)^{16-r}x^{32-5r}$ is a correct simplification in this case.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		(continued)		

Commonly Observed Responses:

COR A

General term has not been isolated

$$\sum_{r=0}^{16}\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$$

$$=\sum_{r=0}^{16}\binom{16}{r}(-1)^r\,2^{16-r}\,x^{32-5r}$$

Do not award •[1]. Award •[2] and •[3].

COR B

General term has been isolated

$$\sum_{r=0}^{16}\binom{16}{r}\left(2x^2\right)^{16-r}\left(-\frac{1}{x^3}\right)^r$$

$$=\binom{16}{r}(-1)^r\,2^{16-r}\,x^{32-5r}$$

Disregard the incorrect use of the final equals sign. Award •[1], •[2] and •[3].

COR C

Binomial expression has been equated to the general term

$$\left(2x^2-\frac{1}{x^3}\right)^{16}=\binom{16}{r}(-1)^r\,2^{16-r}\,x^{32-5r}$$

Disregard the incorrect use of the equals sign. Award •[1].

COR D

Negative sign omitted

$$\binom{16}{r}\left(2x^2\right)^{16-r}\left(\frac{1}{x^3}\right)^r$$

Do not award •[1] or •[3], but •[2] is still available.

COR E

Brackets omitted around -1 in final expression

$$\binom{16}{r}-1^r\,2^{16-r}\,x^{32-5r}$$

Do not award •[3].

COR F

Negative sign has been associated with x in final expression

$$\binom{16}{r}2^{16-r}\left(-x^{32-5r}\right)\text{ or }\binom{16}{r}2^{16-r}\left(-x\right)^{32-5r}$$

Award •[2] for the appearance of x^{32-5r}.
Do not award •[3].

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•4 determine the value of r [1,2] •5 evaluate coefficient [2,3,4]	•4 $r = 10$ •5 512512	**2**

Notes:

1. A candidate starting from $\binom{16}{r}\left(2x^2\right)^r\left(-\frac{1}{x^3}\right)^{16-r}$ should have $r = 6$ at •4.
2. Where a candidate writes out a full expansion, •4 may be awarded only if the expansion is complete and correct at least as far as the required term (in either direction). The required term must clearly identified in the expansion for •5 to be awarded.
3. Where a candidate has omitted $\binom{16}{r}$ in (a), do not award •5, unless it now appears in (b).
4. At •5 accept $\frac{512512}{x^{18}}$.

Commonly Observed Responses:

Binomial expansion

$$65536x^{32} - 524288x^{27} + 1966080x^{22} - 4587520x^{17} + 7454720x^{12} - 8945664x^{7} + 8200192x^{2}$$
$$-5857280x^{-3} + 3294720x^{-8} - 1464320x^{-13} + 512512x^{-18} - 139776x^{-23} + 29120x^{-28}$$
$$-4480x^{-33} + 480x^{-38} - 32x^{-43} + x^{-48}$$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 begin to find $\frac{dy}{dt}$ •2 find $\frac{dy}{dt}$ 1 •3 simplified expression for $\frac{dy}{dx}$ 2	•1 $\frac{dy}{dt} = 4\ln t + \ldots$ **OR** $\frac{dy}{dt} = \ldots + 4t \times \frac{1}{t}$ •2 $\frac{dy}{dt} = 4\ln t + 4$ •3 $\frac{dy}{dx} = \frac{2(\ln t + 1)}{t}$	**3**

Notes:

1. Accept an unsimplified expression for $\frac{dy}{dt}$ for the award of •2.
2. Accept $\frac{dy}{dx} = \frac{2\ln t + 2}{t}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 begin to differentiate $\frac{dy}{dx}$ with respect to t 1 •5 complete differentiation of $\frac{dy}{dx}$ with respect to t 1,2 •6 simplify $\frac{d^2y}{dx^2}$ 1	•4 $\frac{\frac{2}{t}t - \ldots}{t^2}$ or $\frac{\ldots - 2(\ln t + 1)}{t^2}$ •5 $\frac{\frac{2}{t}t - 2(\ln t + 1)}{t^2}$ •6 $\frac{-\ln t}{t^3}$	**3**

Notes:

1. Where a candidate has not attempted to differentiate their answer to (a) with respect to t, award 0/3.
2. Where a candidate produces an incorrect expression in (a), differentiation must involve a product or quotient, including a logarithmic term, for the award of •5.

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		**(continued)**		

Commonly Observed Responses:

Candidate uses a formula method

COR A (starting from unsimplified first derivative)

$\dfrac{\frac{4}{t}\times 2t-\ldots}{(2t)^3}$ or $\dfrac{\ldots-2(4\ln t+4)}{(2t)^3}$ award •4

$\dfrac{\frac{4}{t}\times 2t-2(4\ln t+4)}{(2t)^3}$ award •5

$\dfrac{-\ln t}{t^3}$ award •6

COR B (starting from simplified first derivative)

$\dfrac{\frac{2}{t}\times t-\ldots}{t^3}$ or $\dfrac{\ldots-(2\ln t+2)}{t^3}$ award •4

$\dfrac{\frac{2}{t}\times t-(2\ln t+2)}{t^3}$ award •5

$\dfrac{-2\ln t}{t^3}\times\dfrac{1}{2}$ leading to $\dfrac{-\ln t}{t^3}$ award •6

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(i)	Method 1	Method 1	**2**
			•¹ all three derivatives and all four evaluations	•¹ $f(x)=e^{2x}$ $f(0)=1$ $f'(x)=2e^{2x}$ $f'(0)=2$ $f''(x)=4e^{2x}$ $f''(0)=4$ $f'''(x)=8e^{2x}$ $f'''(0)=8$ stated or implied	
			•² obtain simplified expression [1]	•² $1+2x+2x^2+\frac{4}{3}x^3$	
			Method 2	Method 2	
			•¹ write down Maclaurin series for e^x	•¹ $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ stated or implied	
			•² substitute and simplify [1]	•² $1+2x+2x^2+\frac{4}{3}x^3$	

Notes:

1. Evidence of full simplification may appear in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)	(ii)	Method 1 •³ all three derivatives and all four evaluations [1] •⁴ obtain simplified expression [2,3] Method 2 •³ write down Maclaurin series for $\sin x$ [1] •⁴ substitute and simplify [2,3]	Method 1 •³ $g(x)=\sin 3x$ $\quad g(0)=0$ $g'(x)=3\cos 3x$ $\quad g'(0)=3$ $g''(x)=-9\sin 3x$ $\quad g''(0)=0$ $g'''(x)=-27\cos 3x$ $\quad g'''(0)=-27$ stated or implied •⁴ $3x-\frac{9}{2}x^3$ Method 2 •³ $x-\frac{x^3}{3!}$ stated or implied •⁴ $3x-\frac{9}{2}x^3$	**2**

Notes:

1. Disregard the repeated use of notation from (a) eg $f(x)$.
2. Do not accept $3x+-\frac{9}{2}x^3$ unless resolved in (b), but accept $3x+\frac{-9}{2}x^3$.
3. Evidence of full simplification may appear in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁵ set up composition of expansions [1,2] •⁶ expand and simplify [1,2,3]	•⁵ $1+2\left(3x-\frac{9}{2}x^3\right)+2\left(3x-\frac{9}{2}x^3\right)^2+\frac{4}{3}\left(3x-\frac{9}{2}x^3\right)^3$ •⁶ $1+6x+18x^2+27x^3$	**2**

Notes:

1. For candidates who attempt to multiply expressions from (a), award 0/2.
2. For candidates who attempt an answer from first principles, award 0/2.
3. Disregard higher order terms, whether correct, incorrect or absent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 correct form of integral including limits 1 •2 square y and substitute 1 •3 integrate 2 •4 substitute limits and simplify 3 •5 evaluate 3	•1 $\int_0^a \pi y^2 dx$ •2 $\int_0^a \pi \frac{1}{1+x^2} dx$ •3 $\tan^{-1} x$ •4 $\tan^{-1} a = \frac{\pi}{3}$ •5 $\sqrt{3}$	5

Notes:

1. For the award of •1:
    a. limits must appear at some point
    b. dx must appear at some point.
2. For the award of •3, the integration must be beyond Higher level.
3. Neither •4 nor •5 is available where a candidate produces either:
    a. limits which are both constants
    b. an expression which is not an inverse trigonometric function.

Commonly Observed Responses:

Candidates who use a as their lower limit:

$\int_a^0 \pi y^2 dx$	award •1
$\int_a^0 \pi \frac{1}{1+x^2} dx$	award •2
$\tan^{-1} x$	award •3
$-\tan^{-1} a = \frac{\pi}{3}$	award •4
$-\sqrt{3}$	award •5

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ state expression	•¹ $-3+2d$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² find common difference	•² 4	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•³ substitute [1,2,3]	•³ $500=\frac{n}{2}\left(-6+4\left(n-1\right)\right)$	**3**
			•⁴ write quadratic in standard form and solve [1,4,5]	•⁴ eg $4n^2-10n-1000=0$; 17.1	
			•⁵ communicate result [1,5,6]	•⁵ 18	

Notes:

1. Where a candidate adopts a non-algebraic approach, award 0/3.
2. At •³ accept $500<\frac{n}{2}\left(-6+4\left(n-1\right)\right)$.
3. Evidence for •³ may appear later in the solution, eg by comparing an incorrectly simplified expression with 500.
4. Candidates are not required to give the negative root at •⁴
5. Where a candidate produces a whole number for n at •⁴, •⁵ is available only if they increase this number by one
6. •⁵ is available only for a single positive value derived from attempting to solve a quadratic equation in standard form. Do not accept a range of values.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 interpret $\frac{dV}{dt}$ 1,2	•1 $\frac{dV}{dt} = 12$	4
			•2 state relationship	•2 $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent	
			•3 find $\frac{dV}{dr}$ 3	•3 $\frac{dV}{dr} = 15\pi r^2$	
			•4 evaluate $\frac{dr}{dt}$ 2,4,5,6,7	•4 $\frac{dr}{dt} = \frac{1}{125\pi}$ mm per minute	

Notes:

1. Where a candidate uses a variable other than t (or T) for time (eg "min"), this must be explicitly defined. Otherwise, •1 is not available.
2. Where a candidate converts to centimetres and seconds, $\frac{dV}{dt} = 2 \times 10^{-4}$ and $\frac{dr}{dt} = 4.2 \times 10^{-6}$ cm/s.
3. Where a candidate equates V to $\frac{dV}{dr}$, •3 is not available.
4. Do not accept a negative answer at •4.
5. At •4, candidates must explicitly identify $\frac{dr}{dt}$.
6. Accept an answer rounded or truncated to at least two significant figures.
7. At •4, units must be correct.

Commonly Observed Responses:

Using implicit differentiation

$\frac{dV}{dt} = 15\pi r^2 \times \frac{dr}{dt}$ award •2 and •3.

Question			Generic scheme	Illustrative scheme	Max mark
11.		A	•[1] give counterexample[1] and communicate [1,2,3]	•[1] eg $3^2 + 4^2 = 25$ which is not prime	3
		B	•[2] state form of two consecutive integers [4,5,6]	•[2] eg $k,\ k+1,\ k \in \mathbb{Z}$	
			•[3] show $k^2 + (k+1)^2$ is odd and communicate [7,8]	•[3] $2(k^2 + k) + 1$ which is odd	

Notes:

1. Where the answer relating to statement A contains incorrect information (before, between or after correct information) •[1] is not available.
2. Alternative communication for "not prime" includes "$25 = 5 \times 5$" or "25 is divisible by 5" or "25 is composite."
3. Accept "A is false" for communication at •[1].
4. For •[2], accept "k is an integer" or "let the integers be k …" but do not accept other source sets in words or symbols.
5. Do not accept eg $2k,\ 2k+1$ at •[2], unless the candidate also considers $2k-1,\ 2k$.
6. Where a candidate starts by equating eg $k^2 + (k+1)^2$ to $2m+1$, •[2] is available only if m is not defined as an integer at this stage.
7. For the award of •[3], the candidate must deal with $k,\ k+1$ or both pairs of $2k,\ 2k+1$ and $2k-1,\ 2k$.
8. Accept "B is true" for communication at •[3].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 identify $\bar{z}$	•1 $x - iy$ stated or implied by •2	5
			•2 substitute for $z, \bar{z}$	•2 $(x+iy)^2 + 20(x-iy) - 156 = 0$	
			•3 equate real or imaginary parts 1	•3 $x^2 - y^2 + 20x - 156 = 0$ or $2xy - 20y = 0$	
			•4 solve imaginary part 2,3	•4 $x = 10$	
			•5 use real part to find pair of solutions 2,3,4	•5 $z = 10 \pm 12i$	

Notes:

1. For •3, accept $2ixy - 20iy = 0$.
2. For the award of •4 or •5, the values of x and y must be real.
3. Where, at •4, a candidate obtains a value of y which leads to two values of x, •5 is still available.
4. At •5, accept $x = 10$, $y = \pm 12$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ state expression •² obtain values for A and B	•¹ $\frac{A}{x}+\frac{B}{x+1}$ •² $A=-2$ and $B=2$	**2**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•³ integrate to find "$uv-$" [1,2] •⁴ differentiate to find "$\int u'v\,dx$" [1,2,3] •⁵ obtain full solution [1,2,4]	•³ $\frac{1}{3}xe^{3x}-\ldots$ •⁴ $\ldots\int\frac{1}{3}e^{3x}dx$ •⁵ $\frac{1}{3}xe^{3x}-\frac{1}{9}e^{3x}+c$	**3**

Notes:

1. Where a candidate integrates both functions, award 0/3.
2. Where a candidate differentiates both functions, award 0/3, unless they have communicated the intention to integrate (see COR B).
3. Disregard the omission of dx at •⁴.
4. Disregard the omission of $+c$ at •⁵

Commonly Observed Responses:

COR A

Use of tabular method.

	D	I
$+$	x	e^{3x}
$-$	1	$\frac{1}{3}e^{3x}$
$+$		$\frac{1}{9}e^{3x}$

Award •³ for first three rows, and •⁴ for the final row. Headings may differ.

COR B

Candidate communicates that $v'=e^{3x}$, $v=3e^{3x}$.

Do not award •³ but •⁴ and •⁵ are still available. In the case of •⁵, this may be as a result of integrating correctly the second time, or as a repeated error.

COR C

Candidate chooses to differentiate e^{3x} and integrate x.

Do not award •³ but •⁴ is available. For the award of •⁵, further applications will be needed to arrive at the correct solution.

Question			Generic scheme	Illustrative scheme	Max mark
13.	(c)		•6 state form of integrating factor [1,2,3]	•6 $e^{\int \frac{-2}{x(x+1)}dx}$ stated or implied	**5**
			•7 substitute partial fractions into expression for integrating factor [1,3]	•7 $e^{\int \frac{-2}{x}+\frac{2}{x+1}dx}$	
			•8 find integrating factor in simplified form [1]	•8 $\frac{(x+1)^2}{x^2}$	
			•9 rewrite as integral equation [1,3,4,5,6,7]	•9 $\frac{(x+1)^2 y}{x^2} = \int xe^{3x}dx$	
			•10 integrate RHS and rearrange [1,6,7,8,9]	•10 $y = \frac{x^2}{(x+1)^2}\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c\right)$	

Notes:

1. Where no attempt is made to produce an integrating factor, award 0/5.
2. Where a candidate writes "$P(x) = \frac{-2}{x(x+1)}$" and then writes eg "$e^{\int P(x)}$", •6 may be awarded.
3. At •6, •7 and •9 disregard the omission of dx.
4. For candidates who produce an incorrect integrating factor, •9 may still be available.
5. Accept an unsimplified (or incorrectly simplified) integrand on the RHS for the award of •9.
6. Where incorrect simplification of the RHS occurs before or after •9, •10 is not available.
7. Where a candidate uses a constant integrating factor, •9 and •10 are not available.
8. At •10, +c must be used appropriately.
9. At •10, accept $y = \frac{x^2\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c\right)}{(x+1)^2}$, or equivalent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)	(i)	•[1] state vectors [1]	•[1] $\begin{pmatrix}-1\\2\\-9\end{pmatrix}$, $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	**1**

Notes:

1. Accept vectors written horizontally, eg $(-1, 2, -9)$.

Commonly Observed Responses:

		(ii)	•[2] evidence of strategy for finding normal [1]	•[2] eg $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & 2 & -9\\2 & -1 & 3\end{vmatrix}$	**3**
			•[3] calculate normal [2]	•[3] $\begin{pmatrix}1\\5\\1\end{pmatrix}$	
			•[4] obtain equation [3]	•[4] $x+5y+z=5$	

Notes:

1. Do not award •[2] where the position vectors of A, B or C are used, or if no strategy is evident.
2. Accept any multiple of $\mathbf{n}$ for •[3].
3. Accept an unsimplified equation at •[4].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		•5 state parametric equations of line •6 substitute into LHS of plane equation [1,2] •7 conclusion [2,3,4]	•5 $x = 1+\lambda$ $y = -1-\lambda$ $z = -1+4\lambda$ •6 $1+\lambda+5(-1-\lambda)-1+4\lambda$ •7 $-5 \neq 5$ the equation is inconsistent so the line and plane do not intersect	**3**

Notes:

1. Do not withhold •6 if a candidate includes the RHS of the plane equation.
2. Demonstrating that a particular point on the line does not lie on the plane gains a mark only as part of a solution exemplified in the COR below.
3. Where a candidate produces an incorrect plane equation or incorrect parametric equations, •7 is unavailable if the line intersects the plane but is available otherwise.
4. The minimum requirements for the award of •7 are eg
   a. "$-5 \neq 5$" **and** "do not intersect"
   b. "$-5 = 5$" **and** "inconsistent/not true" **and** "do not intersect"
   c. "$\lambda = \frac{10}{0}$, which is undefined" **and** "do not intersect".

Commonly Observed Response:

Alternative method for •5 and •6:

$\begin{pmatrix}1\\5\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\4\end{pmatrix} = 0$, so the line is parallel to the plane award •5

Substitute eg $(1,-1,-1)$ into LHS of plane equation award •6

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ separate variables and write down integral equation [1,2]	•¹ $\int \frac{dW}{36-W} = \int \frac{dt}{120}$	**5**
			•² integrate LHS [1,3]	•² $-\ln(36-W)$	
			•³ integrate RHS [1,4]	•³ $\frac{1}{120}t + c$	
			•⁴ find constant of integration [1,4,5,6]	•⁴ $c = -\ln 28$	
			•⁵ express W in terms of t [1,4,5,7,8,9]	•⁵ $W = 36 - 28e^{-\frac{1}{120}t}$	

Notes:

1. Where a candidate attempts to integrate an expression involving W with respect to t, award 0/5
2. .•¹ is not available if either $\int dW$ or $\int dt$ is omitted.
3. For the award of •², $-\ln\ldots$ must be present. Accept $-\ln|36-W|$.
4. •³ may be awarded if constant of integration is omitted. However, •⁴ and •⁵ are unavailable if no constant of integration subsequently appears.
5. If the constant of integration is not given as an exact value, award •⁴ only if the answer is correct to two significant figures (-3.3). However, do not award •⁵ if 28 does not appear in expression for W.
6. Where a candidate evaluates a constant after incorrectly rearranging, •⁴ is still available.
7. For the award of,•⁵ $-\ln\ldots$ must be present at •².
8. Accept equivalent answers- eg $W = 36 - \frac{28}{e^{\frac{1}{120}t}}$.
9. Do not award •⁵ for $W = 36 - e^{-\frac{1}{120}t + \ln 28}$ or $W = 36 - e^{\ln 28}e^{-\frac{1}{120}t}$.

Commonly Observed Responses:

Using integrating factor:

Integrating factor $= e^{\frac{1}{120}t}$ award •¹

$e^{\frac{1}{120}t}W = \int \frac{36}{120}e^{\frac{1}{120}t}$ award •²

$e^{\frac{1}{120}t}W = 36e^{\frac{1}{120}t} + c$ award •³

$c = -28$ award •⁴

$W = 36 - 28e^{-\frac{1}{120}t}$ award •⁵

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)		Method 1 ●6 find $\frac{dW}{dt}$ in terms of t ●7 evaluate $\frac{dW}{dt}$ [1,2] Method 2 ●6 evaluate W at $t=67$ and consider $\frac{dW}{dt}$ ●7 evaluate $\frac{dW}{dt}$ [1,2] Method 3 ●6 find $\frac{dW}{dt}$ ●7 evaluate $\frac{dW}{dt}$ [1,2]	Method 1 ●6 $\frac{dW}{dt}=\frac{36-(36-28e^{-\frac{1}{120}t})}{120}$ ●7 $\frac{7}{30}e^{-\frac{67}{120}}$ (kilograms per minute) Method 2 ●6 $W=20$ and $\frac{dW}{dt}=\ldots$ ●7 $0.13\ldots$ (kilograms per minute) Method 3 ●6 $\frac{dW}{dt}=\frac{28e^{-\frac{1}{120}t}}{120}$ ●7 $\frac{7}{30}e^{-\frac{67}{120}}$ (kilograms per minute)	**2**

Notes:

1. At ●7 accept any answer which rounds to 0.13 to two significant figures.
2. At ●7 units need not be given.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(c)		•8 state limit and give justification 1,2,3,4,5,6,7	•8 $L = 36$ AND $e^{-\frac{1}{120}t} \to 0$ as $t \to \infty$ (or $28e^{-\frac{1}{120}t} \to 0$ as $t \to \infty$)	**1**

Notes:

1. There must be clear identification (eg "limit is 36", "36 as…", "… so 36", "…therefore 36") of the limit equalling 36 (or $35.\dot{9}$ but not $35.99\ldots$).
2. Where a candidate uses an expression which does not produce a limit, •8 is not available.
3. Except as indicated in Note 4, do not accept $W = 36$.
4. Where a candidate proceeds from the original differential equation, accept $L = 36$ along with either:
   a. a statement that $\frac{dW}{dt} = 0$ (or a statement that the rate of change would be 0) "when" $W = 36$ (There must be the explicit appearance of $\frac{dW}{dt}$, $\frac{36-W}{120}$, or reference to derivative or rate of change. $W = 36$ may be implied by substitution into the original differential equation) or
   b. an explicit statement that $\frac{dW}{dt} \to 0$ (or rate of change/derivative…) as $W \to 36$.
5. Do not accept $e^{-\frac{1}{120}\infty}\ldots$, or $e^{-\frac{1}{120}t} = 0$ as part of a justification.
6. For consideration of an expression involving an exponential term, there must be reference (in words or symbols) to that term tending to zero as t increases.
7. Disregard incorrect information which does not relate to candidate's justification.

Commonly Observed Responses:

COR A

36 as $\frac{36-36}{120} = 0$, rate of change $= 0$… award •8

COR B

36 as $\frac{36-36}{120} = 0$ do not award •8 (no mention of derivative)

COR C

$\frac{dW}{dt} = \frac{36-36}{120} = 0$, therefore 36 award •8 (justification followed by conclusion)

COR D

$\frac{dW}{dt} = 0$ as $W \to 36$ so $L = 36$ do not award •8 (mixture of $=, \to$ in same statement)

COR E

$L = 36$ as, after this, $\frac{dW}{dt}$ is decreasing do not award •8

COR F

36, as the rate of change would be 0 do not award •8 (no association with W)

[END OF MARKING INSTRUCTIONS]